Electrical Conductivity in General Relativity

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The general relativistic kinetic theory including the effect of a stationary gravitational field is applied to the electromagnetic transport processes in conductors. Then it is applied to derive the general relativistic Ohm's law where the gravitomagnetic terms are incorporated. The total electric charge quantity and charge distribution inside conductors carrying conduction current in some relativistic cases are considered. The general relativistic Ohm's law is applied to predict new gravitomagnetic and gyroscopic effects which can, in principle, be used to detect the Lense-Thirring and rotational fields.

Key words: general relativity, Lense-Thirring effect, electrical conductivity, Ohm's law

1 INTRODUCTION

Kinetic theory in general relativity is treated by a number of authors, and has been reviewed in much more detail by Ehlers[(1)], using the differential calculus on Riemannian manifolds. Recently the growing interest to the general relativistic kinetic theory in plasma appeared[(2)–(4)]. However, the kinetic theory of electrical conductivity in conductors in the presence of gravitational field is not strongly discussed in the literature and its presentation may be of some interest (The formulation of the kinetic theory of electrical conductivity in the astrophysical matter in the flat spacetime can be found, for example, in[(5)]). In this paper we consider the general relativistic kinetic theory needed to determine on microscopic level the influence of a stationary gravitational field on electromagnetic transport phenomena for normal conductors. This approach is interesting, even from purely theoretical point of view, because it involves the interplay of gravitation, electromagnetism, relativity and statistical physics.

The paper is organized in the following way. First, in §2 we consider Boltzmann's kinetic equation in a manifestly covariant form and the distribution function for electron gas in the local equilibrium. Then in the approximation of relaxation time we derive the general-relativistic nonequilibrium distribution function for electron gas in an applied electromagnetic field which is the key element in the construction of microscopic theory of conductivity: the general relativistic Ohm's law is defined by the corresponding average in velocity space. In §3 it is shown that the theoretical project[(6)] on measuring gyroscopic effect in two series connected coil conductor carrying current and rotating in opposite directions has to give the null result. It appears author of the paper[(6)] ignored the surface charges which compensate rotationally induced space charges. Finally, we apply the derived Ohm's law to a few devices that can be used as detectors of gravitomagnetism and rotation.

2 GENERAL RELATIVISTIC KINETIC THEORY OF ELECTRICAL CONDUCTIVITY

General relativistic equations of macroscopic electrodynamics can be derived in two steps. First, a suitable averaging process must be performed so as to obtain the Maxwell equations, being valid at the macroscopic level, from the fundamental Maxwell-Lorentz equations found at the microscopic one

$$e^{\alpha\beta\mu\nu} f_{\beta\mu;\nu} = 0, \qquad f^{\alpha\beta}{}_{;\beta} = \frac{4\pi}{c} j^{\alpha}, \tag{1}$$

which are general covariant in their form, where in the right-hand side - a microscopic four-current density $j^{\alpha}(x)$, and in the left-hand side - the covariant derivative of microscopic tensor of electromagnetic field $f_{\mu\nu}$. Here Greek indices go through $0, 1, 2, 3$, $e^{\alpha\beta\mu\nu}$ is the tensorial expression for the Levi-Civita symbol[(7)].

We shall neglect all the feed back of the electromagnetic field on the metric itself under the well justified assumption that the electromagnetic energy usually can be considered negligible with respect to the mass energy of source of gravitational field. Averaging out the Maxwell-Lorentz equations in the Riemannian background gives

$$e^{\alpha\beta\mu\nu} F_{\beta\mu,\nu} = 0, \qquad F^{\alpha\beta}{}_{;\beta} = \frac{4\pi}{c} < j^{\alpha} >, \tag{2}$$

where $F_{\mu\nu} \equiv < f_{\mu\nu} >$ is averaged macroscopic tensor of electromagnetic field. Operations of averaging out and covariant differentiation are assumed to commutate (although it can not be justified in the general case, see, for example,[(8)]).

Denoting the particles by twice notation ai, where a is number of atom consisting of particles i, we can write the current density $j^{\alpha}(x)$ as

$$j^{\alpha}(x) = \sum_{a,i} q_{ai} \int_{+\infty}^{-\infty} u^{\alpha}_{ai}(\sigma_{ai}) \delta^4(x - z_{ai}(\sigma_{ai})) d\sigma_{ai}, \tag{3}$$

q_{ai} is the charge of particle ai, σ_{ai} its proper time, and u^{α}_{ai} its four-velocity.

Choose now world line z^{α}_{a} with proper parameter σ_a which describes motion of atom as whole. The relative position of a particle in atom

$$s^{\alpha}_{ai} \equiv z^{\alpha}_{ai} - z^{\alpha}_{a} \tag{4}$$

is defined on the hypersurface $\sigma_a = const$: $s^{\alpha}_{ai} u_{\alpha} = 0$.

Substituting expression (4) into (3) and expanding δ- function in powers of s^{α}_{ai} one can obtain

$$j^{\alpha}(x) = \sum_a \int_{+\infty}^{-\infty} d\sigma_a \sum q_i (u^{\alpha}_a + \frac{ds^{\alpha}_{ai}}{d\sigma_a}) \{\delta^4(x - z_a(\sigma_a)) - s^{\rho}_{ai} \nabla_{\rho} \delta^4(x - z_a)\} \ , \tag{5}$$

when only first terms of expansion are kept. First term of expansion called the free current density has the form

$$j^{\alpha}_f(x) = \sum_a q_a \int_{+\infty}^{-\infty} d\sigma_a u^{\alpha}_a(\sigma_a) \delta^4(x - z_a(\sigma_a)). \tag{6}$$

One can show that the last terms of expansion are the divergence of an antisymmetric tensor of polarization and magnetization of atom $m^{\mu\nu}_a$ (as the general relativistic generalization of Kauffmann's method[(9)])

$$j^{\alpha}_{(in)} = \nabla_{\beta} \sum_a \int d\sigma_a m^{\alpha\beta}_a \delta^4(x - z_a), \tag{7}$$

where $m^{\mu\nu}_a = \pi^{\nu}_a \wedge u^{\mu}_a - \mu^{\mu\nu}_a$, $\pi^{\nu}_a = \sum_i q_i s^{\nu}_{ai}$ is the electric dipole moment, $\mu^{\mu\nu}_a = \frac{1}{2} \sum_i q_i s^{\nu}_{ai} \wedge \frac{ds^{\mu}_{ai}}{d\sigma_a}$ is the covariant magnetic dipole moment, $\wedge$ denotes the wedge product, ∇_{β} represents the covariant derivative.

Now we introduce the microscopic dipole density

$$m^{\alpha\beta}(x) = \sum_a \int d\sigma_a m^{\alpha\beta}_a \delta^4(x - z_a), \tag{8}$$

so that current $< j^{\alpha} >$ becomes

$$< j^{\alpha} > = < j^{\alpha}_f + \nabla_{\alpha} m^{\alpha\beta} > . \tag{9}$$

Upon averaging (and denoting macroscopic quantities by capital letters) with the conventional introduction of macroscopic tensor of electromagnetic induction

$$H^{\alpha\beta} = F^{\alpha\beta} - \frac{4\pi}{c} M^{\alpha\beta}$$

one can obtain Maxwell equations

$$e^{\alpha\beta\mu\nu}F_{\beta\mu,\nu}=0, \qquad H^{\mu\nu}{}_{;\nu}=\frac{4\pi}{c}J^{\mu} \tag{10}$$

being valid for an arbitrary medium in general relativity. Thus the standard form of Maxwell equations is indeed valid in the Riemannian background, if suitable care is taken in the definition of macroscopic quantities, that is electric and magnetic moments are incorporated into a general covariant scheme.

Furthermore the electric current J^{α} is still the sum of two terms corresponding to the convection current and to the conduction current $\hat{j}^{\alpha}$, respectively

$$J^{\alpha}=c\rho_0 u^{\alpha}+\hat{j}^{\alpha}, \quad \hat{j}^{\alpha}u_{\alpha}\equiv 0, \tag{11}$$

ρ_0 is the proper density of free electric charges.

The system of differential equations (10) can be closed if and only if the general-relativistic constitutive relations between the inductions and fields, on the one hand, conduction current and field characteristics, on the another hand, through the medium characteristics are given. Thus as a second step material couples which depend on the nature of matter should be defined.

For the special case of conducting media the conduction current is connected with the electromagnetic field tensor $F^{\alpha\beta}$ according to the generalized Ohm law which can be obtained with help of the classical kinetic theory of a gas in the external gravitational field (a special case of such a gas is the conduction electrons in a metal.).

Kinetic theory can be constructed on the basis of invariant distribution function $f(x,u)$ which depends on the coordinates x^{α} and velocities u^{α} and is described by the general-relativistic Boltzmann's kinetic equation for the electron gas[1,10]

$$\frac{\partial f}{\partial x^{\alpha}}u^{\alpha}+\frac{\partial f}{\partial u^{\alpha}}\frac{du^{\alpha}}{d\sigma}=J(x,u). \tag{12}$$

In the general case the collision integral $J(x,u)$ has complicated form and the kinetic equation (12) is the integro-differential one with respect to the distribution function of electrons in the metal. However in the case of the statistical local equilibrium the collision integral vanishes since in that case the growing of entropy is equal to zero[11]

$$S^{\mu}(x)=-\int\frac{d^3u}{u^0}u^{\mu}f(x,u)[lnf(x,u)-1]\ , \tag{13}$$

that is when the distribution function satisfies to the functional expression

$$f(x,u)f(x,u_1) = f(x,u')f(x,u'_1) \ , \tag{14}$$

where u, u_1 and u', u'_1 are velocities of colliding particles before and after the collision, respectively.

Consequently the equilibrium distribution function for the electron gas subjects to the following kinetic equation

$$\frac{\partial f_0}{\partial x^\alpha} u^\alpha - \Gamma^\alpha_{\beta\gamma} u^\beta u^\gamma \frac{\partial f_0}{\partial u^\alpha} = 0 \ , \tag{15}$$

where f_0 is the equilibrium distribution function.

The local equilibrium of particles can take place only in the stationary gravitational field when spacetime allows the existence of the timelike Killing vector

$$\xi_\alpha = \Xi^{1/2} \lambda_\alpha; \qquad (\xi\xi) = -\Xi \ , \tag{16}$$

λ_α is the 4-velocity of the conductor as a whole.

It is known that the symmetry of space-time (the existence of the Killing vector) corresponds to the conservation law. In fact, the quantity

$$\frac{\delta(u\xi)}{\delta\tau} = u^\mu_{;\nu} u^\nu \xi_\mu + \xi_{\mu;\nu} u^\mu u^\nu = 0 \tag{17}$$

is constant along the geodesic line and can be interpreted as conserved energy of particle. Then the equilibrium distribution function takes the form(10)

$$f_0(x,u) = \exp\{\beta + (\xi(x)u)\} \ . \tag{18}$$

Consider now the equilibrium distribution function of the conduction electrons

$$f_0(x,u) = \exp\{\beta + \frac{e}{mc^2}(A(x)\xi) + (\xi u)\} \tag{19}$$

in the presence of electromagnetic field which satisfies the kinetic equation

$$\frac{\partial f_0}{\partial x^\alpha} u^\alpha - \Gamma^\alpha_{\beta\gamma} u^\beta u^\gamma \frac{\partial f_0}{\partial u^\alpha} + \frac{e}{mc^2} F^{\alpha\sigma} u_\sigma \frac{\partial f_0}{\partial u^\alpha} = 0 \ , \tag{20}$$

if the electromagnetic field is stationary i.e. $\pounds_\xi F_{\alpha\beta} = 0$ and a gauge is chosen in a way when $\pounds_\xi A_\alpha = 0$, where $\pounds_\xi$ is the Lie derivative with respect to ξ^α and A_α is the vector potential of electromagnetic field.

The normalization constant β can be chosen in the following form

$$f_0(x,u) = \exp\left\{\frac{\tilde{\mu}(x) + (\xi(x)u)}{K\tilde{T}(x)}\right\} \ , \tag{21}$$

where $\tilde{\mu}(x) = \Xi^{1/2}\mu(x) = \tilde{\zeta} + eA_\alpha\xi^\alpha$ is the related gravitoelectrochemical potential, ζ is the ordinary chemical potential, $T(x)$ is the temperature measured by observer whose 4-velocity field is λ_α. The following condition

$$\tilde{\mu}_{,\alpha} - \frac{\tilde{\mu} - \epsilon}{\tilde{T}(x)}\tilde{T}_{,\alpha} = \zeta_{,\alpha} - w_\alpha\zeta + eE_\alpha - \frac{\tilde{\mu} - \epsilon}{\tilde{T}(x)}\tilde{T}_{,\alpha} = 0 \ , \tag{22}$$

takes place during the thermodynamical equilibrium. Here $\epsilon = -\xi u$ is the Fermi energy, $\Xi^{1/2}_{,\alpha} = -\Xi^{1/2}\lambda_{\alpha;\beta}\lambda^\beta = -\Xi^{1/2}w_\alpha$, w_α is the absolute acceleration of the conductor, E_α is the electric field as measured by observer at rest relative to the conductor. Equation (22) indicates the well-known fact that inner electric field can be induced inside the conductor even in the absence of a current, see, for example[(12)]

Assume that nonequilibrium distribution of velocities differs from the equilibrium one due to the effect of an applied field and relaxate to the equilibrium state exponentially with time t

$$\frac{\partial f}{\partial t} = J = \frac{\partial(f - f_0)}{\partial t} = -\frac{f - f_0}{\tau(x,u)}, \tag{23}$$

where $\tau(x,u)$ is the relaxation time. The solution of the equation (23) has form

$$(f - f_0)_t = (f - f_0)_{t=t_0}\exp(-t/\tau). \tag{24}$$

Assume the deviation caused by external applied field is small, i.e.

$$f - f_0 = f_1 << 1. \tag{25}$$

According to (23) one can now rewrite the kinetic equation (12) in the form

$$\frac{\partial f}{\partial x^\alpha}u^\alpha - \Gamma^\alpha_{\beta\gamma}u^\beta u^\gamma\frac{\partial f}{\partial u^\alpha} + \frac{e}{mc^2}F^{\alpha\sigma}u_\sigma\frac{\partial f}{\partial u^\alpha} + \frac{f_1}{\tau} = 0 \ . \tag{26}$$

In the linear approximaion in expansion of distrubution function we found that the nonequilibrium function for electron gas in a static gravitational field and zero magnetic field is

$$f_1 = \tau\frac{\partial f_0}{\partial\epsilon}\frac{v^\alpha}{\sqrt{1 - \frac{v^2}{c^2}}}\left\{\frac{\partial\tilde{\mu}}{\partial x^\alpha} - \left(\frac{\tilde{\mu}(x) - \epsilon}{\tilde{T}(x)}\right)\frac{\partial\tilde{T}(x)}{\partial x^\alpha}\right\} \ . \tag{27}$$

Inserting (27) into the expression for 4-current j^α one can get the following expression

$$j^\alpha = \frac{2e}{\hbar^3}\int v^\alpha f_1 d^3p =$$

$$\frac{2e^2\tau}{\hbar^3}\left\{\frac{1}{e}\frac{\partial\tilde{\mu}}{\partial x^\alpha} - \frac{1}{e}\left(\frac{\tilde{\mu}(x)-\epsilon}{\tilde{T}(x)}\right)\frac{\partial\tilde{T}}{\partial x^\rho} + F_{\rho\sigma}\lambda^\sigma\right\}\int \frac{v^\alpha v^\rho}{\sqrt{1-\frac{v^2}{c^2}}}\frac{\partial f_0}{\partial\epsilon}d^3p. \tag{28}$$

The the conduction current is aligned along the direction of the electric field. Taking into account this fact one can write (28) as[13]

$$j_\alpha = \sigma\left\{\frac{\partial\tilde{\mu}_e}{\partial x^\alpha} - \left(\frac{\tilde{\mu}(x)-\epsilon}{\tilde{T}(x)}\right)\frac{\partial\tilde{T}}{\partial x^\alpha} + F_{\alpha\sigma}\lambda^\sigma\right\}\ , \tag{29}$$

where $\sigma = \frac{2e^2\tau}{\hbar^3}\int\frac{v^2}{\sqrt{1-\frac{v^2}{c^2}}}\frac{\partial f_0}{\partial\epsilon}d^3p$ is a coefficient of electrical conductivity.

Let us estimate the coefficient of electrical conductivity in the approximation $\frac{v}{c} << 1$. In this case, $p = \sqrt{2\epsilon m}$, $d^3p = 4\pi p^2 dp = 2\pi m^3\sqrt{2\epsilon}$,

$$\sigma = \frac{16\pi e^2 m}{3\hbar^2}M_1, \tag{30}$$

where $M_n = \int_\infty^0 l(\epsilon)\epsilon^n\frac{\partial f_0}{\partial\epsilon}$; $l(\epsilon)$ is a length of free motion of the electrons.

Similarly it is possible to show that the nonequilibrium distribution function for the electron gas in the stationary gravitational field in the presence of magnetic field leads to the general relativistic Ohm's law

$$\begin{aligned} F_{\alpha\beta}u^\beta = \frac{1}{\sigma}j_\alpha + R_H(F_{\nu\alpha} + u_\alpha u^\sigma F_{\nu\sigma})j^\nu + \Xi^{-1/2}\overset{\perp}{\nabla}_\alpha\,\tilde{\zeta}_e \\ -\beta\Xi^{-1/2}\overset{\perp}{\nabla}_\alpha\,\tilde{T} - R_{gg}j^\beta A_{\alpha\beta}\ , \end{aligned} \tag{31}$$

where β is the thermoelectric power, R_H is the Hall constant, $R_{gg} = 2mc/ne^2$ is galvanogoro-magnetic coefficient, u_α is the four-velocity of the conductor as whole, $A_{\beta\alpha} = u_{[\alpha,\beta]} + u_{[\beta}w_{\alpha]}$ is the relativistic rate of rotation, $\overset{\perp}{\nabla}_\alpha$ is the transversal part of the covariant derivative, [...] denotes antisymmetrization.

One can estimate that in the weak field approximation the following relation is valid[13]

$$\overset{\perp}{\nabla}_\alpha\,\tilde{\zeta}_e = Aw_\alpha = a - \gamma M_a c^2/e\ , \tag{32}$$

where $a = mc^2/e$, γ is parameter of order of 1, M_a is atomic mass[13].

3 GYROSCOPIC AND GRAVITOMAGNETIC EFFECTS IN CURRENT CARRYING CONDUCTORS

The charge distribution and total electric charge quantity in the rotating conductors carrying current has been considered in[(6)]. According to this paper a 'gyroscopic' effect is expected to exist for conductor carrying a direct current. That is opposite electric charges to be produced at the different parts of a (constant) current carrying multiturn rotating solenoid provided that coil has circular rings of wire with opposite directions. If the current aligned along the linear velocity of rotation coincide then the conductor takes positive charge and vice versa. On the base of the predicted 'gyroscopic' effect it has been proposed[(6)] to detect an alternating electric current of 'charge exchange' through series-connected current carrying solenoids with opposite windings during the oscillatory motion around their own axis.

Probably the 'gyroscopic' effect in the sense of the paper[(6)] probably could not exist due to the following two reasons: first, the appearance of the surface charge density σ besides the space one ρ_0 has been ignored in[(6)], second, the expressions being valid only in the flat space-time, for example, transmission formulae from the field tensors to observable quantities or the Ohm's law in standard form have been groundlessly used for the calculations in the rotating frame of reference.

Our aim is to examine the theoretical proposal on measuring 'gyroscopic' effect in series-connected two solenoids carrying azimuthal current in opposite directions[(6)] in the rotating frame of reference

$$ds^2 = -(c^2 - \Omega^2 r^2)dt^2 + 2\Omega r^2 d\varphi dt + dr^2 + r^2 d\varphi^2 + dz^2, \tag{33}$$

where Ω is angular velocity of rotation, $x^1 = r, x^2 = \varphi, x^3 = z$ are the cylindrical coordinates.

The general relativistic expression for the proper charge density ρ_0 inside conductors[(7)]

$$\begin{aligned}\rho_0 = {} & \frac{\epsilon\mu R_H}{c} j^2 + \frac{1}{4\pi}\{(\frac{\epsilon}{\sigma} j^\alpha)_{;\alpha} + [\epsilon^2 \mu R_H (\frac{1}{\sigma} j^2 + \Xi^{-1/2} j^\nu \overset{\perp}{\nabla}_\nu (\Xi^{1/2}\zeta_e)) u^\alpha]_{;\alpha} \\ & - \epsilon R_{gg} A_{\alpha\beta} w^\alpha j^\beta + g^{\alpha\beta}(\epsilon R_{gg} j^\nu A_{\alpha\nu})_{;\beta} - \frac{\epsilon}{\sigma} w^\alpha j_\alpha - \epsilon w^\alpha \Xi^{-1/2} \overset{\perp}{\nabla}_\alpha (\Xi^{1/2}\zeta_e) \\ & + g^{\alpha\beta}(\epsilon \Xi^{-1/2} \overset{\perp}{\nabla}_\alpha (\Xi^{1/2}\zeta_e))_{;\beta} + H^{\alpha\beta}[A_{\beta\alpha} + \epsilon\mu R_H w_\alpha j_\beta + (\epsilon\mu R_H j_\alpha)_{,\beta}]\}\end{aligned} \tag{34}$$

has been derived from the Maxwell equations (10) assuming that the constitutive relations between fields and inductions have linear character as well as using the generalized Ohm law (31). Here ϵ and μ are the parameters for the conductor.

In the general case integral[13]

$$Q = -\frac{1}{c}\int (J^{\alpha})_{*} dS_{\alpha} = -\frac{1}{c}\int J^{\alpha} r_{\alpha} dV = \int \rho_v dV, \tag{35}$$

on any hypersurface $dS^{\alpha\beta\gamma}$ is equal to the sum of electric charges whose world-lines intersect this hypersurface. Here element

$$* \, dS_{\sigma} = \frac{1}{3!} dS^{\alpha\beta\gamma} e_{\alpha\beta\gamma\sigma} = \frac{e_{\sigma} - (ek)k_{\sigma}}{\sqrt{1+(ek)^2}} dV = r_{\sigma} dV \tag{36}$$

is dual to the hypersurface built on the triple of vectors $\{\mathbf{k}, \mathbf{m}, \mathbf{n}\}$

$$m_{\alpha} = \frac{e_{\lambda\alpha\mu\nu} e^{\lambda} n^{\mu} k^{\nu}}{\sqrt{1+(ek)^2}}, n_{\alpha} = \frac{e_{\lambda\alpha\mu\nu} e^{\lambda} k^{\mu} m^{\nu}}{\sqrt{1+(ek)^2}}, k^{\alpha} = -(ek)e^{\alpha} + \sqrt{1+(ek)^2} e^{\mu\alpha\rho\nu} e_{\mu} m_{\rho} n_{\nu}, \tag{37}$$

ρ_v is the charge density per unit volume, time-like unit vector r^{α} can be represented in the form

$$r_{\alpha} = e_{\alpha}\sqrt{1 - v^2/c^2} - v_{\alpha}/c, \tag{38}$$

v_{α} is the velocity of observer with respect to object $\{\mathbf{k}, \mathbf{m}, \mathbf{n}\}$, $k_{\alpha} = v_{\alpha}/v$ that is $(ek) = \frac{v/c}{\sqrt{1-v^2/c^2}}$, $v = \sqrt{v_{\alpha} v^{\alpha}}$.

Assume an infinitely long hollow cylindrical (r_1 and r_2 are radii of interior and exterior surfaces) conductor carrying current in azimuthal direction (I is the current per the unit length of the cylinder) is in the rotating frame of reference (33); see Figure 1.

Due to the cylindrical symmetry of the problem Maxwell equations (2) have the following nonvanishing solutions:
in the cavity of the conductor ($0 < r < r_1$)

$$B_z = \frac{4\pi I}{\sqrt{c^2 - \Omega^2 r^2}}, \quad E_r = \frac{\Omega r}{c} \frac{4\pi I}{\sqrt{c^2 - \Omega^2 r^2}};$$

inside conductor ($r_1 \le r \le r_2$)

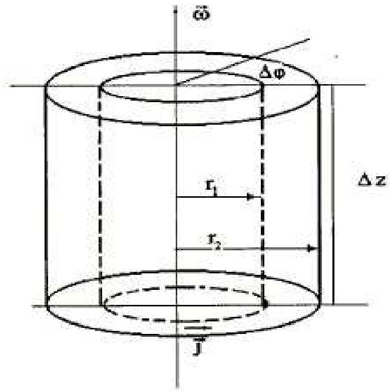

Figure 1: A piece of infinite hollow cylinder carrying current in azimuthal direction in the rotating frame of reference.

$$H_z = \frac{4\pi I}{\sqrt{c^2 - \Omega^2 r^2}} \frac{\ln(r_2/r)}{\ln(r_2/r_1)} \quad , \tag{39}$$

$$E_r = A\frac{\Omega^2 r}{c^2 - \Omega^2 r^2} - \frac{R_{gg} I\Omega}{r(1-\Omega^2 r^2)^2 \ln(r_2/r_1)} \quad , \tag{40}$$

$$\begin{aligned} \rho_0 = \frac{2\Omega c I}{(c^2 - \Omega^2 r^2)^{3/2}} \frac{\ln(r/r_2)}{\ln(r_2/r_1)} + \frac{1}{4\pi}\left\{\frac{\epsilon A \Omega^2 (2c^2 + \Omega^2 r^2)}{(c^2 - \Omega^2 r^2)^2} + \frac{\Omega^2 r}{c^2 - \Omega^2 r^2}\frac{\partial(\epsilon A)}{\partial r}\right\} \\ -\frac{1}{4\pi}\left\{\frac{I\Omega}{r(1-\Omega^2 r^2)^2 \ln(r_2/r_1)}\frac{\partial(R_{gg}\epsilon)}{\partial r} + \frac{5R_{gg}\epsilon I \Omega^3}{(1-\Omega^2 r^2)^3 \ln(r_2/r_1)}\right\} \quad . \end{aligned} \tag{41}$$

Thus according to the generalized Ohm's law (31) and the continuity of the electric current at the boundaries $r = r_1$ and $r = r_2$, the internal radial electric field (40) has two contributions: one is due to the absolute acceleration being proportional to Ω^2 and the second one results from the rotational effect on azimuthal current and is linear in angular velocity of rotation. The nonvanishing space charge (41) generated inside the solenoid is due to three different relativistic reasons: the first term on the left hand side of (41) is produced by the general relativistic effect of charge redistribution[7] in the presence of magnetic field (39), the second one is produced by the inner electric field being proportional to absolute acceleration, and the last one is arising from the interplay between azimuthal electric current and the rotational field.

Densities of surface charges induced at the surfaces r_1 and r_2 of the conductor

$$\sigma_{|r=r_1} = -\frac{\Omega r_1 I}{c\sqrt{c^2-\Omega^2 r_1^2}} + \frac{\epsilon A}{4\pi}\frac{\Omega^2 r_1}{c^2-\Omega^2 r_1^2} - \frac{R_{gg}\epsilon I\Omega}{4\pi r_1(1-\Omega^2 r^2)^2\ln(r_2/r_1)}, \quad (42)$$

$$\sigma_{|r=r_2} = -\frac{\epsilon A}{4\pi}\frac{\Omega^2 r_2}{c^2-\Omega^2 r_2^2} - \frac{R_{gg}\epsilon I\Omega}{4\pi r_1(1-\Omega^2 r^2)^2\ln(r_2/r_1)} \quad (43)$$

have been found from the boundary conditions for the vectors of electromagnetic field.

The total quantity of space charge

$$Q_\rho = \frac{\epsilon A}{4\pi}\left[\frac{\Omega^2 r_2^2/c^2}{(1-\Omega^2 r_2^2/c^2)^{3/2}} - \frac{\Omega^2 r_1^2/c^2}{(1-\Omega^2 r_1^2/c^2)^{3/2}}\right]\Delta z\Delta\varphi + \frac{I\Omega r_1^2}{c^2-\Omega^2 r_1^2}\Delta z\Delta\varphi$$
$$-\frac{R_{gg}\epsilon I\Omega}{4\pi\ln(r_2/r_1)}\left[\frac{1}{(1-\Omega^2 r_2^2)^{5/2}} - \frac{1}{(1-\Omega^2 r_1^2)^{5/2}}\right]\Delta z\Delta\varphi \quad (44)$$

induced inside an arbitrary piece of the solenoid limited by two 2-surfaces $\varphi = const.$ ($\varphi = \varphi_1$ and $\varphi = \varphi_2$, $\Delta\varphi = \varphi_2 - \varphi_1$) and two 2-surfaces $z = const.$ ($z = z_1$ and $z = z_2$, $\Delta z = z_2 - z_1$) is completely compensated by the total quantity of surface charges

$$Q_\sigma = \frac{\epsilon A}{4\pi}\left(\frac{\Omega^2 r_1^2/c^2}{(1-\Omega^2 r_1^2/c^2)^{3/2}} - \frac{\Omega^2 r_2^2/c^2}{(1-\Omega^2 r_2^2/c^2)^{3/2}}\right)\Delta z\Delta\varphi - \frac{I\Omega r_1^2}{c^2-\Omega^2 r_1^2}\Delta z\Delta\varphi$$
$$-\frac{R_{gg}\epsilon I\Omega}{4\pi\ln(r_2/r_1)}\left[\frac{1}{(1-\Omega^2 r_1^2)^{5/2}} - \frac{1}{(1-\Omega^2 r_2^2)^{5/2}}\right]\Delta z\Delta\varphi \quad (45)$$

generated at the pieces of surfaces $r = r_1$ and $r = r_2$.

Thus in spite of violation of the local electric neutrality net sum of the space and surface charges of conductor $Q = Q_\rho + Q_\sigma$ is identically equal to zero and the 'charge exchange' current due to the angular vibration of two series-connected coil conductors carrying azimuthal current in opposite directions(6) does not exist. The suggested proposal(6) on measuring this 'charge exchange' current has to give the null result.

However, we may note that in our recent papers(15) we theoretically predicted the galvanogravitomagnetic and galvanogyroscopic voltages produced by the effect of gravitomagnetic and rotational fields on radial conduction current. Moreover, according to our theoretical results(16), the experiment of Vasil'ev(16) on measurement of vertical magnetic field around rotating hollow neutral cylinder carrying radial thermoelectric current confirms the existence of gyroscopic effects on electric current.

It is naturally to ask the question about a possibility to measure gyroscopic effect in the rotating solenoid carrying azimuthal current. Since, even in the linear in Ω approximation the inner radial electric field $E_r = -R_{gg} I\Omega/r \ln(r_2/r_1)$ is produced inside the solenoid, one can suggest the following non-null proposal on measuring gyroscopic voltage in radial direction. Suppose that the superconducting wire is connected to the inner and outer sides of the solenoid in the way when it forms superconducting loop with SNS junction where the voltage is produced. Then the nonvanishing potential difference V_r would lead to a time varying magnetic flux through the loop. The change in magnetic flux Φ_b inside the circuit during the time interval $[0, t]$ is [18]

$$\Delta\Phi_b = \Delta n \Phi_0 + c \int_0^t V_r dt \ , \tag{46}$$

where $\Phi_0 = \pi\hbar c/e = 2 \times 10^{-7} Gauss \cdot cm^2$ is quantum of the magnetic flux. As long as $\Delta\Phi_b < \Phi_0$, n will remain constant and $\Delta\Phi_b$ will increase linearly with time until $\Delta\Phi_b = \Phi_0$, then the order of the step n will change as flux quantum enters the loop. Thus this particular loop is sensitive to the V_r and in this connection to the angular velocity of rotation.

If the conductor is placed on a platform that is nonrotating relative to the distant stars as determined by telescopes, then, in general, it would rotate with respect to the local inertial frames if a nearby rotating body as the Earth was present because of the general-relativistic Lense-Thirring field which is a consequence of gravitational mass currents[19,20]. If the Earth is approximately spherically symmetric, the Lense-Thirring angular velocity of the local inertial frames relative to the distant stars at position, $\mathbf{r}$, from the centre of the Earth is

$$\mathbf{\Omega}_{LT} = \frac{4GM_\oplus R_\oplus^2}{5c^2} \left[-\frac{\mathbf{\Omega}_\oplus}{r^3} + \frac{(\mathbf{\Omega}_\oplus \mathbf{r}) r}{r^5} \right], \tag{47}$$

where $M_\oplus$, $R_\oplus$, and $\mathbf{\Omega}_\oplus$ are the mass, radius and angular velocity of the Earth, respectively, and G is the gravitational constant. As a rule, the platform on the Earth is rotating with respect to the distant stars, but one can expect that the other effects of rotation may be eliminated with the additional methods.

Assume that a piece of a conductor is at rest on the platform and connected to the source of alternating current in azimuthal direction $i_\varphi = i_\varphi(0) \exp{(i\omega t)}$ with the frequency

ω. Hence, the alternating radial galvanogyroscopic current

$$i_r = i_\varphi(0) \exp(i\omega t) \frac{R_{gg}\sigma\Omega_{LT}}{c} \tag{48}$$

will be produced across the conductor according to the general-relativistic Ohm's law (31). If the amplitude of the supplied azimuthal current is about $10^3 A$, $R_{gg} = 10^{-22}s$, $\sigma = 10^{17}c^{-1}$ the developed azimuthal current has amplitude around $10^{-21}A$. Such currents can be measured, however, in the present case there are serious environmental problems which could reduce the feasibility of the experiment.

4 CONCLUSION

We conclude:

1. The solution of the general-relativistic Boltzmann's kinetic equations for the nonequilibrium distribution function for electron gas leads to the Ohm's law for conduction current being valid in the fields of gravity and inertia. The latter has pure general-relativistic contributions arising from the absolute acceleration and relativistic rate of rotation.
2. 'Gyroscopic' effect in the sense of the paper(6) does not exist since rotationally induced space charges in the solenoid with electric current are totally compensated by the surface charges generated at the inner $r = r_1$ and the outer $r = r_2$ boundaries of the solenoid.
3. Experiment is proposed to measure radial voltage through the rotating solenoid carrying azimuthal electric current.
4. Finally, the nonstationary situation, when alternate azimuthal current produces radial current across the conductor due to the effect of the Earth's Lense-Thirring field has been considered.

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